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Temperature and magnetic field-dependent correlators of the exactly integrable (1 + 1)-dimensional gas of impenetrable fermions

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Abstract. We show that the finite-temperature equal-time field correlators of the exactly integrable (1+1)-dimensional gas of impenetrable fermions in a magnetic field can be expressed as the first Fredholm minor of a completely integrable linear operator. This result enables us to derive differential equations for quantum correlators and to obtain analytical formulae for long-distance asymptotics. In particular, a formula for correlation length as a function of temperature and magnetic field is presented. Our analysis reveals the presence of two length scales, which separate short-distance, intermediate and extreme asymptotic domains. The importance of the cross-over region is emphasized.

1. Introduction

There has been a resurgence of interest lately in the field of (1 + 1)-dimensional exactly integrable models (EIM), following the discovery of numerous connections between this field and other branches of mathematical physics. The theory of knots and links, quantum groups, conformal field theory and two-dimensional gravity are just a few of the recent activities closely related to the analysis of Yang–Baxter algebra which emerged in EIM studies.

The quantum inverse scattering method (QISM) [1] appears to be the most universal tool for the study of a wide class of EIM, providing a general technique for constructing the exact eigenstates of the Hamiltonian and for calculating S-matrices. Recently, important steps towards an eventual solution of the Green function problem by means of the QISM were taken by Smirnov [2] and Its *et al* [3]. A non-trivial generalization of Karowsky's early ideas [4] enabled Smirnov [2] to calculate various form factors of Thirring, sine–Gordon and σ-models. By summing up products of these form factors one might hope to obtain convergent expansions for multipoint Green functions. In [3] Its *et al* deal with Fredholm quantities which arise in the calculation of quantum correlators of the impenetrable Bose gas. They show that Green functions for this model satisfy certain differential equations, and introduce the classical inverse scattering method (CISM) [5] to analyse large-distance asymptotics.

In this paper, inspired by the progress made in [3], we return to our investigation [6] of quantum correlators of the exactly integrable two-component Fermi gas, also known as the fermionic nonlinear Schrödinger model (FNL). Its second quantized Hamiltonian is

$$H = \int_{-\infty}^{\infty} \mathrm{d}x\{\partial_x\psi^\dagger\partial_x\psi + g{:}\psi^\dagger\psi\psi^\dagger\psi{:} - h\psi^\dagger\psi - B\psi^\dagger\sigma_3\psi\} \tag{1.1}$$

where $g > 0$ (repulsive interaction) is the coupling constant, $h > 0$ is the chemical potential, B is the magnetic field and σ_3 is the third component of the Pauli spin matrices. The field operator

$$\psi = \begin{bmatrix} \psi_{+1} \\ \psi_{-1} \end{bmatrix}$$

has two spin components which satisfy the equal-time anticommutation relations

$$\begin{aligned} &\{\psi_i(x), \psi_j^\dagger(y)\} = \delta_{ij}\delta(x-y) \\ &\{\psi_i(x), \psi_j(y)\} = 0 \qquad i, j = 1, -1. \end{aligned} \tag{1.2}$$

The eigenvalue problem for this model was solved by Gaudin [7] and Yang [8] by means of the so-called nested Bethe ansatz technique. Following it up, Lai [9] studied the thermodynamics of the model along the lines of [10]. The QISM analysis of the FNL was initiated by Pu and Zhao [11], who calculated commutation relations between various scattering-data operators and derived quantum Gelfand–Levitan equation for fields ψ_i. In [6] Berkovich and Lowenstein studied field correlators of the FNL in the important case of impenetrable fermions ($g = \infty$). It was proved there that two-point equal-time finite-temperature (T) correlators $\langle \psi_i^\dagger(x)\psi_i(0)\rangle_{T,h,B=0}$ can be representated as the first Fredholm minors of a linear integral operator associated with the kernel

$$K_{T,h}(x) = \tfrac{1}{2} \int \mathrm{d}p \frac{\mathrm{e}^{\mathrm{i}px}}{\frac{1}{2}\mathrm{e}^{(p^2-h)/T} + 1}. \tag{1.3}$$

In the zero-temperature limit, we expressed this first Fredholm minor in terms of the solution of the Painleve-V differential equation and obtained an analytical formula for the infrared asymptotics. The purpose of this paper is to extend this analysis to the more general case of non-zero temperature and magnetic field.

The remainder of this paper is organized as follows. In section 2, we derive an expression for the fields ψ_i in terms of reflection R-operators, and then employ the temperature Green function technique, supplemented by an infrared cut-off procedure, to obtain the Fredholm minor representation of the two-point equal-time correlator. We finish section 2 with the analysis of the limiting case $T = 0$, $B = 0$. Section 3 is dedicated to the general case $T \neq 0$, $B \neq 0$. There we derive differential equations for quantum correlators. In section 4 we made use of the CISM to obtain analytical formulae for the infrared behaviour of the Green functions. We conclude with some remarks about possible generalizations.

2. The Green function as a first Fredholm minor

Our principal goal here is to calculate the two-point equal-time correlation function

$$G_{T,h,B}(x-y) = \frac{\mathrm{Tr}\{\psi_i^\dagger(x)\psi_i(y)\,\mathrm{e}^{-H/T}\}}{\mathrm{Tr}\{\mathrm{e}^{-H/T}\}} \tag{2.1}$$

in this infinite coupling ($g = \infty$) limit. The basic strategy of our QISM approach is to represent $\psi_i^\dagger\psi_i$ as a functional of the so-called reflection $R_j(p)$, $R_j^\dagger(p)$ operators and then use this functional to compute expectation values. The properties of reflection operators [11] which are relevant for the present investigation are

$$(g=\infty)\begin{cases} R_i(p_1)R_j(p_2) = -R_i(p_2)R_j(p_1) \\ R_i(p_1)R_j^\dagger(p_2) = \delta_{ij}(2\pi\delta(p_1-p_2) - \sum_k R_k^\dagger(p_2)R_k(p_1)) \\ i, j, k \equiv \pm 1 \end{cases} \tag{2.2}$$

$$\begin{cases} [H, R_i^\dagger(p)] = (p^2 - h - \mathrm{i}B)R_i^\dagger(p) \\ [P, R_i^\dagger(p)] = pR_i^\dagger(p) \\ [N, R_i^\dagger(p)] = R_i^\dagger(p) \\ [S, R_i^\dagger(p)] = \frac{1}{2}\mathrm{i}R_i(p) \end{cases} \tag{2.3}$$

where P, N, S are momentum, particle number and spin operators, respectively. The eigenstates of H, P, N, S generated by the reflection opegators are quite simple:

$$\begin{gathered} \frac{1}{(2\pi)^{N/2}} R_{j_1}^\dagger(p_1)R_{j_2}^\dagger(p_2)\ldots R_{j_N}^\dagger(p_N)|0\rangle \\ p_N > p_{N-1} > \ldots > p_1. \end{gathered} \tag{2.4}$$

Here $|0\rangle$ is defined by

$$\psi_i(x)|0\rangle = R_i(p)|0\rangle = 0. \tag{2.5}$$

The state (2.4) may be interpreted as a delta-function normalized N-particle in-state with energy $\Sigma_i(p_j^2 - h - j_iB)$, momentum $\Sigma\, p_i$ and spin $\frac{1}{2}\Sigma\, j_i$. Fourier transforming the operators $R_j(p)$,

$$\tilde{R}_j(x) = \frac{1}{2\pi}\int \mathrm{d}p\; \mathrm{e}^{\mathrm{i}px}R_j(p) \tag{2.6}$$

one finds

$$\begin{gathered} \tilde{R}_i(x)\tilde{R}_j(y) = -\tilde{R}_i(y)\tilde{R}_j(x) \\ \tilde{R}_i(x)\tilde{R}_j^\dagger(y) = \delta_{ij}[\delta(x-y) - \sum_k \tilde{R}_k^\dagger(y)\tilde{R}_k(x)]. \end{gathered} \tag{2.7}$$

In analogy with (2.4), $\tilde{R}_j^\dagger(x)$ operators can be employed to construct the complete set of properly normalized in-states:

$$\begin{gathered} \psi_{j_1}^\dagger(x_1)\ldots\psi_{j_N}^\dagger(x_N)|0\rangle = \tilde{R}_{j_1}^\dagger(x_1)\ldots\tilde{R}_{j_N}^\dagger(x_N)|0\rangle \\ x_1 > x_2 > \ldots > x_N. \end{gathered} \tag{2.8}$$

We can now move on to find a representation of the field as a power series in reflection operators. The procedure, proposed in [6], suggests the following ansatz:

$$\begin{aligned} \psi_1(x) = \sum_{m=0}^{\infty}\sum_{l=0}^{m}(-1)^{m-l}\begin{bmatrix} m \\ l \end{bmatrix}\int\prod_{i=1}^{m}\mathrm{d}z_i\,\theta(x > z_1 > \ldots > z_m)\tilde{R}_{j_1}^\dagger(z_1)\ldots\tilde{R}_{j_m}^\dagger(z_m) \\ \times\tilde{R}_{j_m}(z_m)\ldots\tilde{R}_{j_{l+1}}(z_{l+1})\tilde{R}_1(z_l)\tilde{R}_{j_l}(z_{l-1})\ldots\tilde{R}_{j_2}(z_1)\tilde{R}_{j_1}(x). \end{aligned} \tag{2.9}$$

In the formula above, summation over repeated spin indices is understood. To verify (2.9), we compare the action of the LHS of (2.9) on the LHS of (2.8) with that of the RHS of (2.9) on the RHS of (2.8). Making use of the commutation relations (2.2) and (1.2), we conclude after some tedious calculations that these two actions are, in fact, identical. Since states (2.8) form a complete orthornormal basis, this result proves that the ansatz (2.9) is, indeed, the correct representation of the field ψ_j.

The reordering theorem

$$\int_{-\infty}^{y} dz\{\psi_1^\dagger(x), \tilde{R}_j^\dagger(z)\}\tilde{R}_j(z) = 0 \qquad x > y \tag{2.10}$$

proven in [6], enables us to obtain for the bilocal product of fields

$$\begin{aligned}\psi_1^\dagger(x)\psi_1(y) = \sum_{m=0}^{\infty}\sum_{l=0}^{m}\frac{(-1)^l}{m!}\begin{bmatrix} m \\ l\end{bmatrix} \\ \times \int_{-\infty}^{y}\prod_{i=1}^{m} dz_i\, \tilde{R}_{j_1}^\dagger(z_1)\ldots\tilde{R}_{j_m}^\dagger(z_m)\psi_1^\dagger(x)\tilde{R}_{jm}^\dagger(z_m)\ldots\tilde{R}_{j_{l+1}}(z_{l+1}) \\ \times \tilde{R}_1(z_l)\tilde{R}_{j_l}(z_{l-1})\ldots\tilde{R}_{j_2}(z_1)\tilde{R}_1(y) \qquad x > y.\end{aligned} \tag{2.11}$$

If we now expand $\psi_1^\dagger(x)$ inside (2.11) using the ansatz (2.9), we will obtain with the help of (2.6) and (2.7) the representation for $\psi_1^\dagger(x)\psi_1(y)(x > y)$ in terms of normal-ordered products of $R_j(p)$, $R_j^\dagger(p)$ operators:

$$\begin{aligned}\psi_1^\dagger(x)\psi_1(y) = \sum_{\tilde{m},m=0}^{\infty}\sum_{l=0}^{m}\sum_{\tilde{l}=0}^{\tilde{m}}\frac{(-1)^{m+\tilde{m}+l+\tilde{l}}}{m!\,\tilde{m}!}\begin{bmatrix} m \\ l\end{bmatrix}\begin{bmatrix} \tilde{m} \\ \tilde{l}\end{bmatrix}\int\prod_{i=0}^{m+\tilde{m}} dk_i\, dp_i \\ \times\frac{\exp(i\sum_{j=1}^{m}(k_j - p_j)y + i\sum_{j=m+1}^{m+\tilde{m}}(k_j - p_j)x + ik_0 y - ip_0 x)}{(2\pi)^{2(m+\tilde{m}+1)}\prod_{j=1}^{m+\tilde{m}} i(k_j - p_j - i\varepsilon)} \\ \times R_{i_1}^\dagger(p_0)\ldots R_{i_l}^\dagger(p_{l-1})R_1^\dagger(p_l)\ldots R_1^\dagger(p_{m+\tilde{l}})R_{i_{m+\tilde{l}+1}}^\dagger(p_{m+\tilde{l}+1}) \\ \times\ldots R_{i_{m+\tilde{m}}}^\dagger(p_{m+\tilde{m}})R_{i_{m+\tilde{m}}}(k_{m+\tilde{m}})\ldots R_{i_{m+\tilde{l}+1}}(k_{m+\tilde{l}+1}) \\ \times R_1(k_{m+\tilde{l}})\ldots R_1(k_l)R_{i_l}(k_{l-1})\ldots R_{i_1}(k_0) + \text{AS}.\end{aligned} \tag{2.12}$$

Here AS designates contributions of $R^\dagger$, R-operator products with asymmetric spin indices. The explicit expression for AS is not important because its expectation values vanish and thus play no role in Green function calculations.

Before we proceed further, let us comment that the QISM formalism we are using requires that our fields be defined over the entire real line, with boundary conditions at infinity. We therefore expect to encounter infrared divergences while trying to compute traces in (2.1). This infrared problem can be avoided by using the 'infinitesimal boost' regularization procedure [12]:

$$\begin{aligned}G_{T,h,B}(x-y) &= \lim_{q\to 0}\operatorname{Tr}\{\psi_1^\dagger(x)\psi_1(y)\, e^{-H/T}\, e^{-iqM}\} \\ M &= \int dx\, x\sum_{i=\pm 1}\psi_i^\dagger\psi_i;\ e^{iqM}R_i(p)\, e^{-iqM} = R_i(p+q).\end{aligned} \tag{2.13}$$

Equipped with equations (2.12) and (2.13) we are now in a position to calculate $G_{T,h,B}(x-y)$ in a term-by-term fashion, provided that we know how to compute traces or normal-ordered products of reflection operators. The trivial generalization of the trace formula, proved in [6], yields

$$\begin{aligned}&\operatorname{Tr}(R_{i_0}^\dagger(p_0)\ldots R_{i_m}^\dagger(p_m)R_{i_m}(k_m)\ldots R_{i_0}(k_0)\, e^{-H/T}\, e^{-iqM}) \\ &\quad = \sum_{\substack{\{n_j\}=1 \\ 0\le j\le m}}^{\infty}\prod_{j=0}^{m} e^{Bi_j/T}(-e^{B/T} - e^{-B/T})^{n_j - 1}\, e^{-[(p_j^2 - h)n_j]/T} \\ &\quad \times\langle 0|R_{i_m}(k_m)\ldots R_{i_0}(k_0)R_{i_0}^\dagger(p_0 - n_0 q)\ldots R_{i_m}^\dagger(p_m - n_m q)|0\rangle.\end{aligned} \tag{2.14}$$

Finally, substituting (2.12) into (2.13) and making use of the trace formula (2.14) and commutation relations (2.2), we obtain after some lengthy calculations

$$
\begin{aligned}
G_{T,h,B}(x-y) &= \mathrm{e}^{2B/T} \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \mathrm{e}^{-(N+1)B/T} \int \frac{\mathrm{d}k^{N+1}\,\mathrm{d}p^{N+1}}{(2\pi)^{2N+2}} \mathrm{e}^{\mathrm{i}k_0 y - \mathrm{i}p_0 x} \\
&\quad \times \left(\prod_{i=1}^{N} \int_y^x \mathrm{d}z_i\, \mathrm{e}^{\mathrm{i}(k_i - p_i)z_i} \right) \cdot \langle 0 | R_1(k_n) \ldots R_1(k_0) R_1^\dagger(p_0) \ldots R_1^\dagger(p_N) | 0 \rangle \\
&\quad \times \left(\prod_{j=0}^{N} \frac{1}{\mathrm{e}^{(p^2-h)/T} + 2\cosh(B/T)} \right) \\
&= \mathrm{e}^{2B/T} \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \left(\frac{\mathrm{e}^{-B/T}}{2\pi} \right)^{N+1} \\
&\quad \times \int_y^x \mathrm{d}z^N \det \begin{vmatrix} K(x-y) & K(x-z_1) & \ldots & K(x-z_N) \\ K(z_1-y) & & & K(z_1-z_N) \\ \vdots & & & \vdots \\ K(z_N-y) & \ldots & \ldots & K(z_N-z_N) \end{vmatrix} \\
&= \mathrm{e}^{2B/T} D_1(\lambda, x-y)
\end{aligned}
\tag{2.15}
$$

where

$$
K(x) = \int \mathrm{d}p \frac{\mathrm{e}^{\mathrm{i}px}}{\mathrm{e}^{(p^2-h)/T} + 2\cosh(B/T)} \qquad \lambda = \frac{\mathrm{e}^{-B/T}}{2\pi}.
$$

The Green function (2.1) is thus essentially a Fredholm minor $D_1(\lambda, x-y)$ associated with the integral kernel $K(x-y)$. By Kramer's rule, $D_1(\lambda, x-y)$ can be written as the product

$$
D_1(\lambda, x-y) = R(\lambda, x-y) D(\lambda, x-y) \tag{2.16}
$$

where the resolvent $R(\lambda, x-y)$ satisfies the integral equations

$$
(1 - \lambda \hat{K}) R(\lambda, x-y) = \lambda K(x-y) \tag{2.17}
$$

and $D(\lambda, x-y)$ is the Fredholm determinant,

$$
\begin{aligned}
D(\lambda, x-y) &= 1 - \lambda \int_y^x \mathrm{d}z\, K(z,z) + \frac{\lambda^2}{2!} \int_y^x \int_y^x \mathrm{d}z_1\, \mathrm{d}z_2 \begin{vmatrix} K(z_1-z_1) K(z_1-z_2) \\ K(z_2-z_1) K(z_2-z_2) \end{vmatrix} - \ldots \\
&= \det(1 - \lambda \hat{K}).
\end{aligned}
\tag{2.18}
$$

At zero temperature and zero magnetic field, the kernel (2.15) reduces to

$$
K_{T=B=0}(x-y) = \frac{\sin\sqrt{h}(x-y)}{\sqrt{h}(x-y)} \tag{2.19}
$$

and we have for the correlator

$$
G_{T=B=0,h}(x-y) = \sqrt{h}\, D_1\!\left(\lambda = \frac{1}{2\pi}, (x-y)\sqrt{h} \right). \tag{2.20}
$$

Jimbo *et al* [13] have shown that the Fredholm quantities of the integral operator associated with the kernal (2.19) may be expressed in terms of solutions $\phi(\lambda, t)$ of the Painleve-V differential equation

$$\partial_t^2\phi = [(\partial_t\phi)^2 - 1]\cot\phi + \frac{1-\partial_t\phi}{t} \tag{2.21}$$

with the boundary conditions for t tending to zero

$$\phi(t, \lambda) = t - \lambda t^2 + \dots \tag{2.22}$$

In particular, the resolvent $R(1/2\pi, t)$ is

$$R\left(\frac{1}{2\pi}, t\right) = \frac{1-\partial_t\phi}{2\sin\phi} \tag{2.233}$$

while $D(1/2\pi, t)$ satisfies

$$\frac{\partial \ln D(1/2\pi, t)}{\partial t} = \frac{t[(\partial_t\phi)^2 - 1]}{4\sin^2\phi} \qquad D\left(\frac{1}{2\pi}, 0\right) = 1. \tag{2.24}$$

Equations (2.21)-(2.24) specify the correlator (2.20) completely.

Performing the asymptotic analysis of (2.21) along the lines of [12] and [14] yields

$$\phi\left(\frac{1}{2\pi}, t\right) = t - k\ln t - t_0 + \frac{2k\sin[2(t - t_0 - k\ln t)] + 3k^2}{4t} + O\left(\frac{1}{t^2}\right) \tag{2.25}$$

where k, t_0 are integration constants:

$$k = \frac{1}{\pi}\ln 2 \qquad t_0 = k\ln 2 - 2\,\mathrm{Im}\left(\ln\Gamma\left(1 + \mathrm{i}\frac{k}{2}\right)\right). \tag{2.26}$$

Here $\Gamma(1 + \mathrm{i}k/2)$ is the gamma function. Inserting the expansion (2.25) into (2.23) and (2.24) and integrating we obtain

$$G_{T=B=0,h}{}^{(x-y)} \underset{x-y\to\infty}{=} \sqrt{h}\,\frac{k\,\mathrm{e}^{C_0}\,\mathrm{e}^{-k(x-y)\sqrt{h}}}{[(x-y)\sqrt{h}]^{1-(1/2)k^2}}\sin[(x-y)\sqrt{h} - t_0 - k\ln(x-y)\sqrt{h}] \tag{2.27}$$

where C_0 is an integration constant:

$$C_0 = \frac{k^2}{2}(1 + \ln 2) - 2k\,\mathrm{Im}\left(\ln\Gamma\left(1 + \mathrm{i}\frac{k}{2}\right)\right) + 2\,\mathrm{Im}\left(\int_0^k \mathrm{d}\nu\,\ln\Gamma\left(1 + \mathrm{i}\frac{\nu}{2}\right)\right). \tag{2.28}$$

We now go on to the analysis of the finite-temperature and magnetic field case.

3. Partial differential equations for the quantum correlator

In this section we derive a closed form expression for the non-zero temperature and magnetic field two-point correlator (2.15) in terms of the solutions to the partial differential equations which generalize the Painleve-V equation (2.21). To this end, we employ the strategy developed in [3].

Introducing rescaled variables

$$\tilde{x} = \frac{x-y}{2}\sqrt{T} \qquad \tilde{h} = \frac{h}{T} \qquad \tilde{B} = \frac{B}{T} \tag{3.1}$$

and making use of the translational invariance of the FNL, we can rewrite (2.15) as

$$G_{T,\tilde{B}}(\tilde{x},\tilde{h}) = \sqrt{T}\, e^{2\tilde{B}} R(\tilde{x},\tilde{h}) D(\tilde{x},\tilde{h}) \tag{3.2}$$

where $D(\tilde{x},\tilde{h})(R(\tilde{x},\tilde{h}))$ is the Fredholm determinant (resolvent) of the linear integral operator $\hat{K}$ with difference kernel

$$K(\tilde{x}_1-\tilde{x}_2) = \tfrac{1}{2}\int_{-\infty}^{\infty} dp\, \rho(p)\, e^{ip(\tilde{x}_1-\tilde{x}_2)} \qquad \rho(p) = \frac{1}{\frac{1}{2}e^{p^2+\tilde{h}} + \cosh\tilde{B}}. \tag{3.3}$$

For the resolvent $R(\tilde{x},\tilde{h})$ we have

$$\begin{aligned} R(\tilde{x},\tilde{h}) &= \lambda K(\tilde{x}+\tilde{x}) + \lambda^2 \int_{-\tilde{x}}^{\tilde{x}} K(\tilde{x}-z)K(z+\tilde{x})\, dz + \ldots \\ &= \frac{\lambda}{2}\int_{-\infty}^{\infty} dp\, e^{ip\tilde{x}}\sqrt{\rho(p)}\Big(e^{ip\tilde{x}}\sqrt{\rho(p)} + \lambda\int_{-\infty}^{\infty}\sqrt{\rho(p)} \\ &\quad \times \frac{\sin(p-p_1)\tilde{x}}{p-p_1}\sqrt{\rho(p_1)}\, e^{ip_1\tilde{x}}\sqrt{\rho(p_1)}\, dp_1 + \ldots\Big) \\ &= \frac{\lambda}{2}\int_{-\infty}^{\infty} dp\, e^{ip\tilde{x}}\sqrt{\rho(p)}\left(\sum_{k=0}^{\infty}(\lambda\hat{N})^k(\sqrt{\rho(p)}\, e^{ip\tilde{x}})\right) \\ &= \frac{\lambda}{2}\int_{-\infty}^{\infty} dp\, E_+(p) f_+(p) \end{aligned} \tag{3.4}$$

where

$$E_\pm(p) = \sqrt{\rho(p)}\, e^{ip\tilde{x}} \tag{3.5}$$

$$f_\pm(p) = \frac{1}{1-\lambda\hat{N}} E_\pm(p) \tag{3.6}$$

$$\lambda = \frac{e^{-\tilde{B}}}{2\pi} \tag{3.7}$$

and $\hat{N}$ is a linear operator, defined by its action on any function $\Phi(p)$ as follows:

$$\begin{aligned} \hat{N}\Phi(p) &= \int_{-\infty}^{\infty} N(p,p_1)\Phi(p_1)\, dp_1 \\ N(p,p_1) &= \sqrt{\rho(p)}\,\frac{\sin(p-p_1)\tilde{x}}{p-p_1}\sqrt{\rho(p_1)} = \frac{E_+(p)E_-(p_1) - E_-(p)E_+(p_1)}{2i(p-p_1)}. \end{aligned} \tag{3.8}$$

Analogously, for the Fredholm determinant $D(\tilde{x},\tilde{h})$, we can obtain

$$D(\tilde{x},\tilde{h}) = \det(1-\lambda\hat{K}) = \det(1-\lambda\hat{N}). \tag{3.9}$$

Taking advantage of the famous Fredholm identity

$$\ln\det(1-\lambda\hat{N}) = -\sum_{k=1}^{\infty}\frac{\lambda^k}{k}\operatorname{Tr}(\hat{N}^k) \tag{3.10}$$

and of the definitions (3.6) and (3.8), we have for the derivative $\partial_{\tilde{x}}(\ln D(\tilde{x},\tilde{h}))$

$$\begin{aligned} \partial_{\tilde{x}}\ln D(\tilde{x},\tilde{h}) &= -\lambda\operatorname{Tr}\left(\frac{1}{1-\lambda\hat{N}}\partial_{\tilde{x}}\hat{N}\right) = -\lambda\operatorname{Tr}\left(\frac{1}{1-\lambda\hat{N}}E_+E_-\right) \\ &= -\lambda\operatorname{Tr}(f_+E_-) = -\lambda\int dp f_+(p)E_-(p) = -V_{+-}(\tilde{x},\tilde{h}). \end{aligned} \tag{3.11}$$

Similar, although somewhat lengthier calculations for the derivative $\partial_{\tilde{h}} \ln D(\tilde{x}, \tilde{h})$ yield

$$\partial_{\tilde{h}} \ln D(\tilde{x}, \tilde{h}) = -\tilde{x}\partial_{\tilde{h}} V_{+-} + \tfrac{1}{2}(\partial_{\tilde{h}} V_{+-})^2 - \tfrac{1}{2}(\partial_{\tilde{h}} V_{++})^2. \tag{3.12}$$

Here $V_{\pm,\pm}(\tilde{x}, \tilde{h})$ functions are defined as follows:

$$\begin{aligned} V_{+-} &= \lambda \int_{-\infty}^{\infty} \mathrm{d}p f_+(p) E_-(p) = V_{-+} = \lambda \int_{-\infty}^{\infty} \mathrm{d}p f_-(p) E_+(p) \\ V_{++} &= \lambda \int_{-\infty}^{\infty} \mathrm{d}p f_+(p) E_+(p) = V_{--} = \lambda \int_{-\infty}^{\infty} \mathrm{d}p f_-(p) E_-(p). \end{aligned} \tag{3.13}$$

Finally, collecting the results (3.2), (3.4) and (3.11), we obtain the representation for the field correlator (2.1) in terms of the potentials $V_{+,\pm}$:

$$G_{T,\tilde{B}}(\tilde{x}, \tilde{h}) = \sqrt{T}\,\tfrac{1}{2}\,\mathrm{e}^{2\tilde{B}} V_{++}(\tilde{x}, \tilde{h}) \exp\left(- \int_0^{\tilde{x}} V_{+-}(z, \tilde{h})\,\mathrm{d}z \right). \tag{3.14}$$

We now turn to the derivation of partial differential equations for the temperature and magnetic field-dependent correlation function. To this end, we introduce the two-component vector function

$$F(p) = \begin{vmatrix} f_+(p) \\ f_-(p) \end{vmatrix} \tag{3.15}$$

and apply operators $\partial_{\tilde{x}}$ and $(2p\partial_{\tilde{h}} + \partial_p)$ to equation (3.6) to arrive at

$$\begin{aligned} \partial_{\tilde{x}} F &= (\mathrm{i}p\sigma_3 + \sigma_1 V_{++})F = \hat{L}F \\ (2p\partial_{\tilde{h}} + \partial_p)F &= [\mathrm{i}(x - \partial_{\tilde{h}} V_{+-})\sigma_3 - (\partial_{\tilde{h}} V_{++})\sigma_2]F = \hat{M}F. \end{aligned} \tag{3.16}$$

The $\hat{L}$, $\hat{M}$ matrices constitute the famous Lax pair, with p playing the role of spectral parameter. The compatibility (flatness) condition for the overconstrained system (3.16)

$$[\partial_{\tilde{x}} - \hat{L}, 2p\partial_{\tilde{h}} + \partial_p - \hat{M}] = 0 \tag{3.17}$$

leads to the following nonlinear differential equations for potentials $V_{+,\pm}$:

$$\partial_{\tilde{x}} V_{+-} = V_{++}^2 \qquad \partial_{\tilde{h}}(V_{++}^2) = 1 + \partial_{\tilde{x}}\left(\frac{\partial_{\tilde{x}}\partial_{\tilde{h}} V_{++}}{2V_{++}} \right). \tag{3.18}$$

The functions V_{++} and V_{+-} are completely specified by (3.18) along with the boundary conditions

$$\begin{aligned} &V_{++} = \int_{-\infty}^{\infty} \lambda\rho(p, \tilde{h})\,\mathrm{d}p + \tilde{x}\left(\int_{-\infty}^{\infty} \lambda\rho(p, \tilde{h})\,\mathrm{d}p \right)^2 + \mathrm{O}(\tilde{x}^2) \qquad \text{as } \tilde{x} \to 0 \\ &V_{++}(\tilde{x}, h \to -\infty) = 0 \qquad V_{+-}(0, \tilde{h}) = \int_{-\infty}^{\infty} \lambda\rho(p, \tilde{h})\,\mathrm{d}p \end{aligned} \tag{3.19}$$

which follows from the series expression (3.4).

Remarkably, formulae (3.18) and (3.19) differ from analogous formulae, obtained in [3] for the impenetrable bosons, only in the substitution

$$(\lambda\rho)_F = \frac{\mathrm{e}^{-\tilde{B}}}{2\pi} \left(\frac{1}{2}\mathrm{e}^{p^2 - \tilde{h}} + \cosh \tilde{B} \right)^{-1} \to (\lambda\rho)_B = \frac{2}{\pi}(\mathrm{e}^{p^2 - \tilde{h}} + 1)^{-1}. \tag{3.20}$$

Nevertheless, given the sensitivity of the differential equations (3.18) to the initial data, this difference has non-trivial consequences.

The results (3.18) and (3.19), and (3.14), enables us to obtain various asymptotic expansions for the field correlator (2.1). The equations (3.18) and (3.19) are particularly well suited to study the short distance asymptotics ($x \to 0$) and to construct the low-density expansion ($t \to -\infty$). The most interesting problem from the physical point of view is the long distance asymptotics ($x \to \infty$). To be able to make use of the partial differential equations (3.18) in this case, we have to impose boundary conditions at infinity. In other words, one has to have the main term of long-distance asymptotics of (2.1) from the start. This vital piece of information can be obtained by using the matrix Riemann problem (MRP) of the CISM [5] for the classical exactly integrable system (3.16).

Let us introduce two 2×2 matrix-valued functions $\chi^{\pm}(p)$ of complex argument p, which solve the Riemann problem,

$$\chi^{-}(p)=\chi^{+}(p)G(p)\ (\operatorname{Im} p=0) \qquad \det \chi^{\pm}(p)\neq 0 \tag{3.21}$$

with the usual normalization

$$\chi^{\pm}(p)=I+\mathrm{O}\left(\frac{1}{p}\right) \tag{3.22}$$

as $|p|\to\infty$. The functions $\chi^{+}(p)$ and $\chi^{-}(p)$ may be analytically extended into the upper and lower half-planes, respectively. The conjugating matrix $G(p)$ here is

$$G(p)=I+\pi\lambda\begin{vmatrix} E_{+}(p)E_{-}(p) & -E_{+}^{2}(p) \\ E_{-}^{2}(p) & -E_{-}(p)E_{+}(p)\end{vmatrix}. \tag{3.23}$$

To show that the MRP (3.21) is equivalent to the integral equation

$$(1-\lambda\hat{N})F(p)=\begin{vmatrix} E_{+} \\ E_{-}\end{vmatrix}(p) \tag{3.24}$$

for $F(p)$, we observe that the singular integral equation

$$\chi^{+}(p)=I+\frac{1}{2\pi \mathrm{i}}\int\frac{\chi^{+}(p_1)(I-G(p_1))}{p_1-p-\mathrm{i}\varepsilon}\,\mathrm{d}p_1 \tag{3.25}$$

for this problem has the solution

$$\chi^{+}(p)=\begin{bmatrix} 1+\dfrac{\lambda}{2\mathrm{i}}\displaystyle\int_{-\infty}^{\infty}\frac{f_{+}(p_1)E_{-}(p_1)}{p-p_1+\mathrm{i}\varepsilon}\,\mathrm{d}p_1 & -\dfrac{\lambda}{2\mathrm{i}}\displaystyle\int_{-\infty}^{\infty}\frac{f_{+}(p_1)E_{+}(p_1)}{p-p_1+\mathrm{i}\varepsilon}\,\mathrm{d}p_1 \\ \dfrac{\lambda}{2\mathrm{i}}\displaystyle\int\frac{f-(p_1)E_{-}(p_1)}{p-p_1+\mathrm{i}\varepsilon}\,\mathrm{d}p_1 & 1-\dfrac{\lambda}{2\mathrm{i}}\displaystyle\int\frac{f_{-}(p_1)E_{+}(p_1)}{p-p_1+\mathrm{i}\varepsilon}\,\mathrm{d}p_1\end{bmatrix}. \tag{3.26}$$

Then it is easy to verify the identity

$$F(p)=\chi^{+}(p)\begin{vmatrix} E_{+} \\ E_{-}\end{vmatrix}(p) \tag{3.27}$$

which establishes the connection between the MRP (3.21) and the integral equation (3.24). Formula (3.26) allows us to obtain the potentials V_{++} and V_{+-} by taking the $p\to\infty$ limit of the $\chi^{+}(p)$-function:

$$\chi^{+}(p)\underset{p\to\infty}{=}I-\frac{1}{2p}(V_{++}\sigma_2+\mathrm{i}V_{+-}\sigma_3)+\mathrm{O}\left(\frac{1}{p^2}\right)$$

$$\lim_{p\to\infty}2p[I-\chi^{+}(p)]=\sigma_2V_{++}+\mathrm{i}\sigma_3V_{+-}. \tag{3.28}$$

Thus, we see that the asymptotic behaviour of the potentials $V_{+,\pm}$ and the field correlator (3.2) can be understood in terms of the asymptotical properties of the matrix-valued function $\chi(p)$. Fortunately, these properties are studied in great detail [5].

4. Asymptotics

In this section, we derive the main terms of the asymptotics at potentials V_{++}, V_{+-} and of the field correlator (3.2). The complete asymptotical expansion can then be obtained from the partial differential equations (3.18). To make contact with the extremely useful results of [5], we transform the conjugating matrix $G(p)$ (3.23) into the regular one, with diagonal matrix elements equal to unity. To this end, let us introduce the transformation

$$\chi^{\pm}(p) \Rightarrow \tilde{\chi}^{\pm}(p) = \chi^{\pm}(p)\begin{bmatrix} \alpha^{\pm}(p) & 0 \\ 0 & \beta^{\pm}(p) \end{bmatrix} \tag{4.1}$$

where $\alpha^{+}(p)$, $\beta^{+}(p)$ and $\alpha^{-}(p)$, $\beta^{-}(p)$ are some holomorhpic functions for $\operatorname{Im} p > 0$ and $\operatorname{Im} p < 0$, respectively, and they tend to unity in the limit $|p| \to \infty$. The MRP (3.21) can now be rewritten as

$$\tilde{\chi}^{-}(p) = \tilde{\chi}^{+}(p)\tilde{G}(p)(\operatorname{Im} p = 0)$$

$$\tilde{G}(p) = \left[\begin{array}{c|c} (1+\pi\lambda\rho(p))\dfrac{\alpha^{-}(p)}{\alpha^{+}(p)} & -\pi\lambda\rho(p)\dfrac{\beta^{-}(p)}{\alpha^{+}(p)} \\ \hline \pi\lambda\rho(p)\dfrac{\alpha^{-}(p)}{\beta^{+}(p)} & (1-\pi\lambda\rho(p))\dfrac{\beta^{-}(p)}{\beta^{+}(p)} \end{array}\right]. \tag{4.2}$$

Imposing the requirement that diagonal matrix elements of $\tilde{G}(p)$ are equal to one, we have

$$\frac{\alpha^{-}(p)}{\alpha^{+}(p)} = [1+\pi\lambda\rho(p)]^{-1} \qquad \operatorname{Im} p = 0 \tag{4.3a}$$

$$\frac{\beta^{-}(p)}{\beta^{+}(p)} = [1-\pi\lambda\rho(p)]^{-1} \qquad \operatorname{Im} p = 0. \tag{4.3b}$$

It is easy to verify that the functions

$$\begin{aligned} \alpha^{\pm}(p) &= \exp\left(\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty} \mathrm{d}p_1 \frac{1}{p_1 - p \mp \mathrm{i}\varepsilon} \ln(1+\pi\lambda\rho(p_1))\right) \\ \beta^{\pm}(p) &= \exp\left(\frac{1}{2\pi_\mathrm{i}}\int_{-\infty}^{\infty} \mathrm{d}p_1 \frac{1}{p_1 - p \mp \mathrm{i}\varepsilon} \ln(1-\pi\lambda\rho(p_1))\right) \end{aligned} \tag{4.4}$$

are solutions of the corresponding scalar Riemann problems (4.3a) and (4.3b). They also satisfy normalization conditions at infinity:

$$\lim_{|p|\to\infty} \alpha^{\pm}(p) = \lim_{|p|\to\infty} \beta^{\pm}(p) = 1. \tag{4.5}$$

Inserting (4.3) into (4.2), we finally have

$$\tilde{G}(p) = \begin{pmatrix} 1 & \bar{\varepsilon} b^{*}(p)\,\mathrm{e}^{\mathrm{i}2p\bar{x}} \\ -b(p)\,\mathrm{e}^{-\mathrm{i}2p\bar{x}} & 1 \end{pmatrix}, \ \bar{\varepsilon} = 1, \qquad \operatorname{Im} p = 0 \tag{4.6}$$

with functions $b(p)$, $b^*(p)$ given by

$$b(p)=-\pi\lambda\rho(p)\frac{\alpha^-(p)}{\beta^+(p)} \qquad b^*(p)=-\pi\lambda\rho(p)\frac{\beta^-(p)}{\alpha^+(p)}. \tag{4.7}$$

We now list the properties of the MRP (4.2), proven in [5], which are relevant to the present investigation:

(i) The asymptotic behaviour of $\tilde{G}_{11}(p)$ for real p, as $x\to\infty$,

$$\tilde{G}_{11}(p)\xrightarrow[x\to\infty]{} a^{-1}(p) \tag{4.8}$$

where $a^{-1}(p)$ (the transmission coefficient) is given by

$$a^{-1}(p)=\exp\left(\frac{-1}{2\pi \mathrm{i}}\int_{-\infty}^{\infty}\frac{\mathrm{d}p_1}{p_1-p-\mathrm{i}\varepsilon}\log(1+b^*(p_1)b(p_1))\right). \tag{4.9}$$

Note that since $\tilde{\varepsilon}=1$, there are no Blaschke factors in the formula above.

Taking into account (4.4), we have for $a^{-1}(p)$:

$$a^{-1}(p)=\exp\left(\frac{1}{2\pi \mathrm{i}}\int_{-\infty}^{\infty}\frac{\mathrm{d}p_1}{p_1-p-\mathrm{i}\varepsilon}\log(1-\pi^2\lambda^2\rho^2(p_1))\right)=\alpha^+(p)\beta^+(p). \tag{4.10}$$

(ii) The classical counterpart of the expansion (2.9) is

$$V_{++}(\tilde{x})=r(\tilde{x})+\mathrm{O}(r^2(\tilde{x})). \tag{4.11}$$

Here the reflection coefficient $r(x)$ is defined as

$$r(\tilde{x})=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{b^*(p)}{a(p)}\,\mathrm{e}^{\mathrm{i}2p\tilde{x}}\,\mathrm{d}p. \tag{4.12}$$

Making use of (4.7), (4.9) and (4.3b), we have

$$r(\tilde{x})=2\lambda\int_{-\infty}^{\infty}(\beta^+(p))^2\frac{\mathrm{e}^{\mathrm{i}2p\tilde{x}}}{\mathrm{e}^{p^2-\tilde{h}}+\mathrm{e}^{\tilde{B}}}\,\mathrm{d}p. \tag{4.13}$$

From (3.28), (4.1), (4.4), (4.8) and (4.10) it follows that the main term in the long-distance asymptotics of potential $V_{+-}(x)$ can be represented as

$$V_{+-}(x)\underset{x\to\infty}{=}-\frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}p\,\ln(1-\pi\lambda\rho(p)). \tag{4.14}$$

To obtain a similar expression for the potential $V_{++}(x)$, let us observe that the integration contour in the formula (4.13) can be shifted in the upper half-plane and the integral, therefore, can be replaced by the sum

$$r(\tilde{x})=\mathrm{e}^{-2\tilde{B}}\sum_{k=0}^{\infty}\left((\beta^+(p_k))^2\frac{\mathrm{e}^{\mathrm{i}2p_k\tilde{x}}}{\mathrm{i}p_k}-(\beta^+(-p_k^*))^2\frac{\mathrm{e}^{-\mathrm{i}2p_k^*\tilde{x}}}{\mathrm{i}p_k^*}\right). \tag{4.15}$$

Here p_k's, defined as zeros of the function $(\mathrm{e}^{p^2-\tilde{h}}+\mathrm{e}^{\tilde{B}})$, are

$$\begin{aligned}\sqrt{2}p_k&=\{[(\tilde{h}+\tilde{B})^2+\pi^2(2k+1)^2]^{1/2}+(\tilde{h}+\tilde{B})\}^{1/2}\\&\quad+\mathrm{i}\{[(\tilde{h}+\tilde{B})^2+\pi^2(2k+1)^2]^{1/2}-(\tilde{h}+\tilde{B})\}^{1/2}.\end{aligned} \tag{4.16}$$

Clearly, it is contributions due to the roots p_0, $-p_0^*$ that will determine the main term in the asymptotic expansion for $r(\tilde{x})$ as $\tilde{x}\to\infty$. Thus, we have for the potential $V_{++}(\tilde{x})$ (4.11):

$$
\begin{aligned}
V_{++}(\tilde{x}) &= r(\tilde{x}) + \mathrm{O}(\mathrm{e}^{-4(\operatorname{Im} p_0)\tilde{x}}) \\
&= 2\,\mathrm{e}^{-2\tilde{B}}\operatorname{Re}\left((\beta^{+}(p_0))^2\frac{\mathrm{e}^{\mathrm{i}2p_0\tilde{x}}}{\mathrm{i}p_0}\right) \\
&\quad + \begin{cases} \mathrm{O}(\mathrm{e}^{-4(\operatorname{Im} p_0)\tilde{x}}) & \text{if } \sqrt{\frac{49}{240}} \leqslant (\tilde{h}+\tilde{B})/\pi \\ \mathrm{O}(\mathrm{e}^{-2(\operatorname{Im} p_1)\tilde{x}}) & \text{if } \sqrt{\frac{49}{240}} \geqslant (\tilde{h}+\tilde{B})/\pi. \end{cases}
\end{aligned} \tag{4.17}
$$

This concludes our derivation of the main terms (4.14) and (4.17) in the asymptotic expansions for the potentials V_{++}, V_{+-}. Let us now apply these results to get the asymptotics of $\ln D(\tilde{x}, \tilde{h})$. From equations (3.11), (3.12), (4.14) and (4.17) it follows that

$$
\ln D(\tilde{x}\to\infty, \tilde{h}) = -\tilde{x}u(\tilde{h}) + \tfrac{1}{2}\int_{-\infty}^{\tilde{h}} \mathrm{d}y(\partial_y u(y))^2 + \ln C_1. \tag{4.18}
$$

Here

$$
u(y) = \frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}p\,\ln\left[\frac{\mathrm{e}^{p^2-y}+2\cosh\tilde{B}}{\mathrm{e}^{p^2-y}+\mathrm{e}^{\tilde{B}}}\right] \tag{4.19}
$$

and C_1 is some numerical constant.

Finally, we have for the two-point field correlator (3.2)

$$
\begin{aligned}
G_{T,B}(x-y) &= C_1\sqrt{T}\operatorname{Re}\left((\beta^{+}(p_0))^2\frac{\exp[\mathrm{i}(x-y)\sqrt{T}p_0]}{\mathrm{i}p_0}\right) \\
&\quad\times\exp\left(-\frac{x-y}{2}\sqrt{T}u(\tilde{h}) + \int_{-\infty}^{\tilde{h}}\mathrm{d}y\frac{(\partial_y u)^2}{2}\right) \\
&= K\exp(x-y/\xi)\sin(p_+(x-y)+\tilde{\phi})
\end{aligned} \tag{4.20}
$$

where

$$
\begin{aligned}
K = \frac{C_1 T}{[(h+B)^2+\pi^2T^2]^{1/4}}\exp\Bigg[\frac{1}{2}\int_{-\infty}^{\tilde{h}}\mathrm{d}y(\partial_y u)^2 - \frac{p_-}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{(p-p_+)^2+p_-^2} \\
\times\ln\left(1+\frac{\exp(-2B/T)}{\exp[(p^2-h-B)/T]+1}\right)\Bigg]
\end{aligned} \tag{4.21a}
$$

$$
\tilde{\phi} = \frac{1}{\pi}\int\frac{\mathrm{d}p(p-p_+)}{(p-p_+)^2+p_-^2}\ln\left[1+\frac{\mathrm{e}^{-2B/T}}{\mathrm{e}^{(p^2-h-B)/T}+1}\right] - \tan^{-1}\frac{p_-}{p_+} \tag{4.21b}
$$

$$
\frac{1}{\xi} = p_- + \frac{1}{\pi}\int_0^{\infty}\mathrm{d}p\,\ln\left[1+\frac{\mathrm{e}^{-2B/T}}{\mathrm{e}^{(p^2-h-B)/T}+1}\right] \tag{4.21c}
$$

and

$$
p_{\pm} = \frac{1}{\sqrt{2}}\{[(h+B)^2+\pi^2T^2]^{1/2}\pm(h+B)\}^{1/2}. \tag{4.21d}
$$

Formula (4.21c) gives the temperature and magnetic field dependence of the correlation length of impenetrable fermions. In the limit of small temperature we have

$$\xi=\begin{cases}\dfrac{\pi}{\sqrt{h+B}}\ln^{-1}(1+e^{-2B/T}) & h+B>0,\ T\to 0,\ \dfrac{B}{T}=\text{finite}\\[2mm] \dfrac{1}{\sqrt{h+B}} & h+B<0,\ T\to 0,\ \dfrac{B}{T}=\text{finite.}\end{cases} \tag{4.22}$$

In the high-temperature limit the correlation length becomes

$$\xi=\frac{1}{\sqrt{T}}\left[\sqrt{\frac{\pi}{2}}+\frac{1}{\pi}\int_0^\infty \mathrm{d}p\ \ln\left(\frac{2+e^{p^2}}{1+e^{p^2}}\right)\right]^{-1}. \tag{4.23}$$

In order to find the scale at which the asymptotics (4.20) sets in, let us recall that in deriving (4.17) we kept only the first term and neglected the second one in the expansion. This is a valid procedure, provided that

$$x-y\gg x_0=\begin{cases}\dfrac{1}{p_-} & \text{if } \pi T\sqrt{\frac{49}{240}}\leqslant(h+B)\\[2mm] \dfrac{1}{p_- -\sqrt{T}\,\mathrm{Im}\,p_1} & \text{if } h+B\leqslant \pi T\sqrt{\frac{49}{240}}.\end{cases} \tag{4.24}$$

In general, the two scales x_0 and ξ are quite different. Indeed, let us consider the $B=0$, $T\ll h$ case. From (4.21d) and (4.22) it follows that

$$x_0=\frac{2\sqrt{h}}{\pi T}\gg\xi=\frac{\pi}{(\ln 2)\sqrt{h}}. \tag{4.25}$$

Before we move on, let us point out that the simple presence of the two original scale parameters $\sqrt{h}$, T does not automatically imply that the two scales ξ, x_0, defining three different regions

$$\begin{array}{lll}(\mathrm{I})\ 0\leqslant x-y\ll\sqrt{h} & \text{(short distance)} & \\ (\mathrm{II})\ \sqrt{h}\ll x-y\ll x_0 & \text{(intermediate domain)} & (4.26)\\ (\mathrm{III})\ x_0\ll x-y & \text{(long distance)} & \end{array}$$

would remerge in the analysis of Green function behaviour. The formula (4.25) suggests that asymptotic domain III shrinks to zero as $T\to 0$. If one, nevertheless, tries to impose the $T\to 0$ limit in (4.20) and (4.21), one finds that the phase (4.21b) blows up:

$$\tilde{\phi}(T\to 0)\sim\ln\frac{h}{T}\xrightarrow[T\to 0]{}\infty. \tag{4.27}$$

Miraculously, the formula for the correlation length (4.21c) survives this limiting procedure to yield the correct result:

$$\lim_{T\to 0}\xi(B=0,\ T\to 0)=\xi(B=0,\ T=0)=\frac{\pi\ln^{-1}(2)}{\sqrt{h}}. \tag{4.28}$$

As T tend sto zero, we may interpret the result (2.27) as a formula for the intermediate asymptotics with domain of validity II. The cross-over region ($x\approx x_0$), which smoothly connects the intermediate II and extreme asymptotic III domains, is the most difficult to analyse. Note now that the two-point field correlator becomes very small at distances

$x \gg x_0$. The cross-over region thus contains the bulk of information about the Green function, where this function is significantly different from zero. As a final remark, we comment that this information can be exploited to determine the overall constant C_1 in the formula (4.18). The author will report on some results pertaining to the cross-over behaviour in a future publication.

5. Concluding remarks

We anticipate it will be straightforward to calculate time-dependent multipoint correlators for the impenetrable fermions using expansion (2.9) as a convenient starting point. Such a calculation is presently underway, as is an attempt to extend our treatment to the case of the most general matrix NLM, describing a mixture of fermions and bosons, interacting via a (infinite) delta-function potential. the generalization to the finite-coupling case will be much more difficult, since a non-trivial modification of the Fredholm determinant representation must be found before any further progress can be made. As far as application of the CISM to analysis of infrared asymptotics is concerned, we believe that the most stubborn unresolved problem is that of the undetermined constant C_1 (4.18). Note that even in the much simpler case of the Painleve-V equation ((2.21) and (2.22)) ($T=0$) this problem is not solved [15].

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